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Electric current and conductance in a chain of quantum dots

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Abstract

By using the non-equilibrium Green function method, we give the current flowing in a chain of quantum dots exactly. The result coincides with the Landauer formula. Finally, the conductance of the system is given in the linear approximation.

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1. Introduction

The quantum feature of the conductance in microscopic electronics is currently a major theoretical and experimental research topic in condensed matter physics [1]. With technique development, devices on a smaller and smaller scale are needed, in which the quantum effects will become a problem. One subject is to study the properties of the electronic transport in the quantum dot, which has many interesting phenomena, such as the Coulomb blockade, resonant tunnelling [2] and the Kondo effect [3–5]. The experimental device is constructed by a nanoparticle coupled to two leads generally. These leads may be metal or superconductor [6,7] and the nanoparticle is usually treated as a quantum dot. In past papers, as we know, one simple quantum dot [8] or double quantum dot [9,10] is considered, in which there may be one energy level or two energy levels. These give nonlinear current through this system.

In this paper we study a single-tunnelling-channel model in which N quantum dots interacting with each other construct a one-dimensional chain between two mental leads. Due to this interaction in the system, we could not have the aid of the conventional Laudauer theory to give the transport current. Lending to the non-equilibrium Green method and the Languish theory [11], we derive the current and the conductance in the quantum chain. It is seen that the result of the current coincides with the Landauer formula in the non-interacting case [12].

The paper is arranged as follows. In section 2, the Hamiltonian for a chain of quantum dots is given. By choosing a suitable rotational transformation, this chain can be transferred to a series of parallel quasi-particles, which connect with the reservoirs isolatively. In section 3, a general current flowing in the lead is given. The static current and the conductance are get in section 4. Finally, we give a conclusion in section 5.

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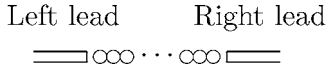

Figure 1. The chain of quantum dots.

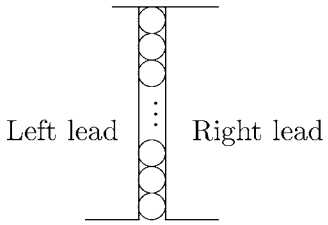

Figure 2. The chain of quantum dots after the rotational transformation.

2. Hamiltonian for a chain of quantum dots

Neglecting the electronic spin component and the Coulomb interaction of the dot, i.e. the four-fermion interacting terms in the Hubbard model have been omitted, the Hamiltonian of the system is written as

$$H = \sum_{\alpha=L,R} H_\alpha + H_d + \sum_{\alpha=L,R} H_{T\alpha} \tag{1}$$

where

$$\begin{aligned} H_\alpha &= \sum_{k\in\alpha} \epsilon_k C_k^+ C_k \qquad \alpha = L, R \\ H_d &= \sum_{i=1}^{n} W_i d_i^+ d_i + \sum_{i=1}^{n-1} [V_i d_i^+ d_{i+1} + \text{h.c.}] \\ H_{TL} &= \sum_{k\in L} [V_k^L C_k^+ d_1 + \text{h.c.}] \qquad H_{TR} = \sum_{k\in R} [V_k^R C_k^+ d_n + \text{h.c.}] \end{aligned} \tag{2}$$

where C_k^+ (C_k) and d_I^+ (d_I) are the creation and annihilation operators for the lead and quantum dots. H_α and H_d are the Hamiltonians for the two reservoirs and the chain of the quantum dots respectively. $H_{T\alpha}(\alpha = L, R)$ describes the interaction between the two leads with the dots at the two end of the chain, by which the current can flow in the system. V_k^α are the coupling constants, which may generally depend on time. The physics of this device is shown in figure 1. In order to give the transport current, a rotational transformation $d_i = \sum_{j=1}^{n} T_{ij} a_j$ is needed, in which we have introduced n new fermionic operators a_j to express the quasi-particles (quasi-dots). By choosing a proper transformation, the Hamiltonian H_d becomes diagonal, that is

$$H_d' = \sum_{i=1}^{n} E_i a_i^+ a_i. \tag{3}$$

In the meantime, the interacting Hamiltonian $H_{T\alpha}$ becomes

$$H_{T\alpha}' = \sum_{k\in\alpha, i=1}^{n} [U_{ki}^\alpha C_k^+ a_i + \text{h.c.}] \tag{4}$$

where the coupling constants become $U_{ki}^L = V_k^L T_{1i}$ and $U_{ki}^R = V_k^R T_{ni}$. The Hamiltonians for the two reservoirs maintain their initial forms under the unitary transformation. The Hamiltonian describing the system is then

$$H = \sum_{\alpha=L,R} H_\alpha + H_d' + \sum_{\alpha=L,R} H_{T\alpha}'. \tag{5}$$

It is seen from equation (4) that each quasi-particle described by operator a_i interacts with the two leads isolatively. This is similar to one multi-level quantum dot system acting on the two reservoirs. The physical scheme becomes figure 2 for the quasi-dots. It must be noted that the commutation relation maintains its initial form $[a_i, a_j^+]_+ = \delta_{ij}$. It seems that there is no coupling between these quasi-particles, which is not in fact true. The coupling between the quasi-particle and the reservoir results indirectly in coupling among the quasi-particles, so the current flowing through this system is not equal to the sum of the currents flowing in every quasi-particle. It is determined in the following way.

The current in the system is calculated by the time derivation of the number operator $N_\alpha = \sum_{k\in\alpha} C_k^+ C_k$ ($\alpha = L, R$). Lending to the Heisenberg equation, we first give the current which follows in the left-hand lead:

$$\begin{aligned} J_L &= e\left\langle \frac{\mathrm{d}N_L}{\mathrm{d}t} \right\rangle = \frac{\mathrm{i}e}{\hbar}\langle [H, N_L] \rangle \\ &= \frac{e}{\hbar} \sum_{k\in L, j=1}^{n} [U_{kj}^L G_{jk}^<(t,t) - U_{kj}^{L*} G_{kj}^<(t,t)] \end{aligned} \tag{6}$$

where we have defined the Green functions $G_{jk}^<(t,t') = \mathrm{i}\langle C_k^+(t') a_j(t)\rangle$ and $G_{kj}^<(t,t') = \mathrm{i}\langle a_j^+(t') C_k(t)\rangle$. By using the property $G_{kj}^{<*}(t,t) = -G_{jk}^<(t,t)$, the current is written as

$$J_L = \frac{2e}{\hbar}\mathrm{Re} \sum_{k\in L, j=1}^{n} U_{kj}^L G_{jk}^<(t,t). \tag{7}$$

As the same process, the current flowing in the right-hand lead is derived from the right-hand lead's number operator

$$J_R = \frac{2e}{\hbar}\mathrm{Re} \sum_{k\in R, j=1}^{n} U_{kj}^R G_{jk}^<(t,t). \tag{8}$$

In the static condition, the current is continuous, i.e. $J_L = -J_R$. So the current through the system can be expressed as

$$\begin{aligned} J &= \tfrac{1}{2}(J_L - J_R) \\ &= \frac{e}{\hbar}\mathrm{Re}\left[\sum_{k\in L, j=1}^{n} U_{kj}^L G_{jk}^<(t,t) - \sum_{k\in R, j=1}^{n} U_{kj}^R G_{jk}^<(t,t) \right]. \end{aligned} \tag{9}$$

3. The current in the non-static case

Equation (9) shows that the Green function $G_{jk}^<(t,t')$ plays an important part in the calculation of the current. Our next aim is to give its expression. For simplicity, we select $\hbar = 1$ in the following. First we define the Green function for the left-hand lead as $G_{jk}(t,t') = -\mathrm{i}\langle T a_j(t) C_k^+(t')\rangle$, where $k \in L$ (it should be noted that if $k \in R$, then $G_{jk}(t,t')$ gives the function for the right-hand lead). By lending to the Heisenberg equation, $G_{jk}(t,t')$ satisfies the following kinetic equation:

$$\left(\mathrm{i}\frac{\partial}{\partial t'} + \epsilon_k\right) G_{jk}(t,t') = -\sum_{m=1}^{n} U_{km}^{L*} G_{jm}(t,t') \tag{10}$$

where we have defined the Green function $G_{jm}(t,t') = -\mathrm{i}\langle T a_j(t) a_m^+(t')\rangle$ for the chain. This function links only with the operator of the quantum dots. In order to solve the

above equation, we need further to define the Green function for the uncoupled lead as $g_k(t,t') = -\mathrm{i}\langle TC_k(t)C_k^+(t')\rangle$. As the same process, we have

$$\left(\mathrm{i}\frac{\partial}{\partial t'} + \epsilon_k\right) g_k(t,t') = -\delta(t-t'). \tag{11}$$

In other respects, the Green functions for the uncoupled lead are calculated straightforwardly; they are

$$\begin{aligned} g_k^<(t,t') &= \mathrm{i}\langle C_k^+(t')C_k(t)\rangle = \mathrm{i}f(\epsilon_k)\exp\left[\mathrm{i}\int_{t'}^{t}\mathrm{d}t''\,\epsilon_k(t'')\right] \\ g_k^+(t,t') &= -\mathrm{i}\langle C_k(t)C_k^+(t')\rangle = -\mathrm{i}(1-f(\epsilon_k))\exp\left[\mathrm{i}\int_{t'}^{t}\mathrm{d}t''\,\epsilon_k(t'')\right] \\ g_k^{r,a}(t,t') &= \mp\mathrm{i}\theta(\pm t\mp t')\langle[C_k^+(t),C_k(t')]_+\rangle = \mp\mathrm{i}\theta(\pm t\mp t')\exp\left[\mathrm{i}\int_{t'}^{t}\mathrm{d}t''\,\epsilon_k(t'')\right] \end{aligned} \tag{12}$$

where $f(\epsilon_k)$ $(k\in\alpha)$ is the particle distribution function of the αth reservoir. Leading to equation (11), the solution of equation (10) is

$$G_{jk}(t,t') = \int \mathrm{d}t_1 \sum_{m=1}^{n} U_{km}^{L*} G_{jm}(t,t_1) g_k(t_1-t'). \tag{13}$$

It is easy to write the Green function $G_{kj}(t,t') = -\mathrm{i}\langle TC_j(t)a_j^+(t')\rangle$

$$G_{kj}(t,t') = \int \mathrm{d}t_1 \sum_{m=1}^{n} g_k(t-t_1) U_{km}^{L} G_{mj}(t_1,t'). \tag{14}$$

In order to give a series of Green functions, we need to obtain the expression of the Green function in the time course by referring to equation (14)

$$G_{jk}(\tau,\tau') = \int \mathrm{d}\tau_1 \sum_{m=1}^{n} U_{km}^{L*} G_{jm}(\tau,\tau_1) g_k(\tau_1-\tau') \tag{15}$$

where the τ_1-integral is over the time loop. By using the Langreth operator rule [13], we have

$$\begin{aligned} G_{jk}^<(t,t') &= \sum_{m=1}^{n}\int \mathrm{d}t_1\, U_{km}^{L*}[G_{jm}^r(t,t_1)g_k^<(t_1-t') + G_{jm}^<(t,t_1)g_k^a(t_1-t')] \\ G_{jk}^r(t,t') &= \sum_{m=1}^{n}\int \mathrm{d}t_1\, U_{km}^{L*} G_{jm}^r(t,t_1) g_k^r(t_1-t'). \end{aligned} \tag{16}$$

Due to the uncoupling Green function $g_k^{\langle,\rangle,r,a}$ being known, our next destination is to determine the Green function $G_{jm}(t,t') = -\mathrm{i}\langle Ta_j(t)a_m^+(t')\rangle$ in order to give the concrete expression $G_{jk}(t,t')$. Taking equation (10), we have

$$\left(\mathrm{i}\frac{\partial}{\partial t} - E_j\right) G_{jm}(t,t') = \delta(t-t')\delta_{jm} + \sum_{k\in\alpha,\alpha=L,R} U_{kj}^{\alpha*} G_{km}(t,t'). \tag{17}$$

In other respects, the Green functions for the uncoupling quantum dots are $g_{jm}(t,t') = -\mathrm{i}\langle Ta_j(t)a_m^+(t')\rangle$, which satisfy the following kinetic equations:

$$\left(\mathrm{i}\frac{\partial}{\partial t} - E_j\right) g_{jm}(t,t') = \delta(t-t')\delta_{jm}, \tag{18}$$

but the concrete result for the uncoupling Green functions $g_{jm}(t,t')$ can be calculated straightforwardly; these are

$$g_{jm}^{r,a}(t,t') = \mp\mathrm{i}\theta(\pm t\mp t')\langle[a_j^+(t),a_m(t')]_+\rangle = \mp\mathrm{i}\theta(\pm t\mp t')\exp\left[\mathrm{i}\int_{t'}^{t}\mathrm{d}t''\,E_j(t'')\right]. \tag{19}$$

Using equation (18), equation (17) has the form

$$G_{jm}(t,t') = g_{jm}(t-t') + \sum_{p=1;k\in\alpha;\alpha=L,M} \int \mathrm{d}t_1\, g_{jp}(t-t_1) U_{kp}^{\alpha *} G_{km}(t_1,t') \tag{20}$$

which shows that although these quasi-dots after transformation are decoupled, there is still correlation through the reservoirs. Combining equation (14) with (20), we have

$$G_{jm}(t,t') = g_{jm}(t-t') + \sum_{p=1;q=1}^{n} \int \mathrm{d}t_1\, \mathrm{d}t_2\, g_{jp}(t-t_1) \sum_{\boldsymbol{p},\boldsymbol{q}} (t_1,t_2) G_{qm}(t_2,t') \tag{21}$$

where $\sum_{\boldsymbol{p},\boldsymbol{q}}^{r,a,\langle,\rangle}(t_1,t_2) = \sum_{k\in\alpha;\alpha=L,R} U_{kp}^{\alpha *} g_k^{r,a,\langle,\rangle}(t_1,t_2) U_{kq}^{\alpha}$ is the self-energy. This shows that the function $\sum_{\boldsymbol{pq}}^{r,a,\langle,\rangle}(t_1,t_2)$ is known due to $U_{kp}^{\alpha *}$ and $g_k^{r,a,\langle,\rangle}$ having been given. So the current is calculated with equations (9), (16) and (21), but (21) is a reiterating equation. By now the current of the time-dependent system can be obtained, if we combine equations (9), (13) with (21).

4. The current in the static case

Our next aim is to discuss the time-independent coupling system in the spectral representation, i.e. ϵ_k, E_j and W_i, V_j in equation (2) do not depend on time. Due to the time being translation invariant, these uncoupling Green functions of the system are time difference $(t-t')$ functions

$$\begin{aligned} g_k^{<}(t-t') &= \mathrm{i} f(\epsilon_k) \exp[\mathrm{i}\epsilon_k(t-t')] \\ g_k^{+}(t-t') &= -\mathrm{i}(1-f(\epsilon_k)) \exp[\mathrm{i}\epsilon_k(t-t')] \\ g_k^{r,a}(t-t') &= \mp \mathrm{i}\theta(\pm t \mp t') \exp[\mathrm{i}\epsilon_k(t-t')] \end{aligned} \tag{22}$$

and

$$g_{jm}^{r,a}(t-t') = \mp \mathrm{i}\theta(\pm t \mp t') \exp[\mathrm{i}E_j(t-t')].$$

The corresponding spectral functions for the uncoupling Green functions, then, are

$$\begin{aligned} g_k^{<}(\omega) &= \mathrm{i}2\pi f(\epsilon)\delta(\omega-\epsilon_k) \\ g_k^{>}(\omega) &= -\mathrm{i}2\pi(1-f(\epsilon))\delta(\omega-\epsilon_k) \\ g_k^{r,a}(\omega) &= \frac{1}{\omega-\epsilon_k \mp \mathrm{i}\delta} \\ g_{jm}^{r,a}(\omega) &= \frac{\delta_{jm}}{\omega-E_j \mp \mathrm{i}\delta}. \end{aligned} \tag{23}$$

It is seen from equations (23) that due to the translation invariance of the uncoupling Green function, the Green function $G_{jm}(t,t')$ is also translation invariant, i.e. it can be written as $G_{jm}(t-t')$. Moreover, the function $\sum_{\boldsymbol{pq}}^{r,a,\langle,\rangle}(t_1,t_2)$ has the same property. So the spectral representation of the self-energy function $\sum_{\boldsymbol{pq}}^{<}(t_1-t_2)$ is

$$\begin{aligned} \sum_{pq}^{<}(\omega) &= \mathrm{i}2\pi \sum_{k\in\alpha,\alpha=L,R} f_\alpha(\epsilon_k) U_{kp}^{\alpha *} U_{kq}^{\alpha} \delta(\omega-\epsilon_k) \\ &= \mathrm{i}2\pi \sum_{\alpha=L,R} f_\alpha(\omega) U_p^{\alpha *}(\omega) U_q^{\alpha}(\omega) \rho_\alpha(\omega) \end{aligned} \tag{24}$$

where $\rho_\alpha(\omega)$ is the state density of the leads and we have denoted $U_{kp}^{\alpha} = U_p^{\alpha}(\omega)$ when $\omega = \epsilon_k$. Here the electron distribution functions in the two reservoirs have been clearly written as $f_\alpha(\omega)$. Defining $\Gamma_{pq}^{\alpha}(\omega) = 2\pi\rho_\alpha(\omega) U_p^{\alpha *}(\omega) U_q^{\alpha}(\omega)$, then we have

$$\sum_{pq}^{<}(\omega) = \mathrm{i}[\Gamma_{pq}^{L}(\omega) f_L(\omega) + \Gamma_{pq}^{R}(\omega) f_R(\omega)]. \tag{25}$$

Similarly the retard and advance self-energy functions $\sum_{pq}^{r,a}(\omega)$ are

$$\sum_{pq}^{r,a}(\omega) = \Lambda_{pq}(\omega) \pm \frac{\mathrm{i}}{2}\Gamma_{pq}(\omega) \tag{26}$$

with $\Lambda_{pq}(\omega) = \sum_{\alpha=L,R} \Lambda_{pq}^{\alpha}(\omega)$, $\Gamma_{pq}(\omega) = \sum_{\alpha=L,R} \Gamma_{pq}^{\alpha}(\omega)$ and

$$\Lambda_{pq}^{\alpha}(\omega) = \int \mathrm{d}\epsilon\, \rho(\epsilon) U_p^{\alpha *}(\epsilon) U_q^{\alpha}(\epsilon) P\left(\frac{1}{\omega}\right).$$

From equation (23), the Green function in the spectral representation is

$$G_{jm}(\omega) = g_{jm}(\omega) + \sum_{pq=1}^{n} g_{jp}(\omega) \sum_{pq}(\omega) G_{qm}(\omega) \tag{27}$$

or it can be simply written as

$$G_{jm}^{-1}(\omega) = g_{jm}^{-1}(\omega) - \sum_{pq}(\omega). \tag{28}$$

In other respects, using Langreth theory, the retard and advance Green functions $G_{jm}^{r,a}(\omega)$ can be expressed as

$$G_{jm}^{r,a}(\omega) = g_{jm}^{r,a}(\omega) + \sum_{pq=1}^{n} g_{jp}^{r,a}(\omega) \sum_{pq}^{r,a}(\omega) G_{qm}^{r,a}(\omega). \tag{29}$$

It is necessary for us to give another expression for the retard and advance Green functions

$$\begin{aligned} G_{jm}^{r,a}(\omega) &= \left[g_{jm}^{r,a-1}(\omega) - \sum_{pq}^{r,a}(\omega) \right]^{-1} \\ &= \frac{1}{(\omega - E_j)\delta_{jm} - \Lambda_{jm}(\omega) \pm \frac{\mathrm{i}}{2}\Gamma_{jm}(\omega)}. \end{aligned} \tag{30}$$

Now, we only give the spectral representation of $G_{jm}(\omega)$. According to the Langreth rule, we can derive other Green functions by using the above equation:

$$G_{jm}^{<}(\omega) = \sum_{pq=1}^{n} G_{jp}^{r}(\omega) \sum_{pq}^{<}(\omega) G_{qm}^{a}(\omega). \tag{31}$$

Combining equations (16) and (30), the spectral representation of the Green function $G_{jk}^{<}(\omega)$ is

$$G_{jk}^{<}(\omega) = \sum_{m=1}^{n} U_{km}^{\alpha *}[G_{jm}^{r}(\omega) g_k^{<}(\omega) + G_{jm}^{<}(\omega) g_k^{a}(\omega)]. \tag{32}$$

Inserting the spectral representation of the uncoupling Green function equation (23) into (32), we have

$$G_{jk}^{<}(\omega) = \sum_{m} U_{km}^{L*} \left[\mathrm{i}2\pi f_L(\epsilon_k)\delta(\omega) G_{jm}^{r}(\omega) + \frac{1}{\omega - \epsilon_k - \mathrm{i}\delta} G_{jm}^{<}(\omega) \right]. \tag{33}$$

Now many necessary mathematical results have been given. The next destination is to express the current in the spectral form. Considering the translation invariance of the system, equation (6) can be expressed as in the spectral representation

$$J_L = \frac{e}{\hbar} \sum_{j=1}^{n} \frac{1}{2\pi} \int \omega [U_j^{L}(\omega) G_{jk}^{<}(\omega) - U_j^{L*}(\omega) G_{kj}^{<}(\omega)]. \tag{34}$$

Lending to equations (33) and (34), the current in the left-hand lead can be written as

$$J_L = \frac{e}{\hbar}\sum_{m,j=1}^{n}\frac{1}{2\pi}\int \mathrm{d}\omega\, U_j^{L*}(\omega)U_m^{L*}(\omega)[g_k(\omega)(G^r_{jm}(\omega) - G^a_{jm}(\omega)) + (g_k^<(\omega) - g_k^>(\omega))G^<_{jm}(\omega)]. \tag{35}$$

By using the concrete expression of the uncoupling Green function $g_k(\omega)$, the current flowing in the left-hand lead is

$$J_L = \frac{\mathrm{i}e}{2\pi\hbar}\sum_{m,j=1}^{n}\int \mathrm{d}\epsilon\, \Gamma^L_{jm}(\epsilon)[f_L(\epsilon)(G^r_{jm}(\epsilon) - G^a_{jm}(\epsilon)) + G^<_{jm}(\epsilon)]. \tag{36}$$

Similarly the current flowing in the right-hand lead is

$$J_R = \frac{\mathrm{i}e}{2\pi\hbar}\sum_{m,j=1}^{n}\int \mathrm{d}\epsilon\, \Gamma^R_{jm}(\epsilon)[f_R(\epsilon)(G^r_{jm}(\epsilon) - G^a_{jm}(\epsilon)) + G^<_{jm}(\epsilon)]. \tag{37}$$

It is necessary for us to give the concrete form of the function $G^r_{jm}(\epsilon) - G^a_{jm}(\epsilon)$. From equation (30), we have

$$G^r_{jm}(\epsilon) - G^a_{jm}(\epsilon) = \frac{-\mathrm{i}\Lambda_{jm}(\omega)}{[(\omega - E_j)\delta_{jm} - \Gamma_{jm}(\omega)]^2 + \frac{1}{4}\Gamma^2_{jm}(\omega)}. \tag{38}$$

Combining equation (9) with (36) and (37), the current flowing through the system is written as in the spectral expression

$$J = \frac{\mathrm{i}e}{4\pi\hbar}\sum_{m,j=1}^{n}\int \mathrm{d}\epsilon\{[\Gamma^L_{jm}(\epsilon)f_L(\epsilon) - \Gamma^R_{jm}(\epsilon)f_R(\epsilon)](G^r_{jm}(\epsilon) - G^a_{jm}(\epsilon)) + [\Gamma^L_{jm}(\epsilon) - \Gamma^R_{jm}(\epsilon)]G^<_{jm}(\epsilon)]\}. \tag{39}$$

Another expression of the current is derived in the following way. From equations (25) and (31), we obtain

$$G^<_{jm}(\omega) = \sum_{\alpha=L,R}\sum_{p,q=1}^{n} f_\alpha(\omega)G^r_{jp}(\omega)\Gamma^\alpha_{pq}(\omega)G^a_{qm}(\omega). \tag{40}$$

Equation (29) can be expressed in the other form ${G^r_{jm}}^{-1}(\omega) = {g^r_{jm}}^{-1}(\omega) - \sum^r_{mj}(\omega)$, and ${G^a_{jm}}^{-1}(\omega) = {g^a_{jm}}^{-1}(\omega) - \sum^a_{mj}(\omega)$, so we have

$${G^r_{jm}}^{-1}(\omega) - {G^a_{jm}}^{-1}(\omega) = \sum_{jm}^{a}(\omega) - \sum_{jm}^{r}(\omega). \tag{41}$$

Finally, the current is written as by using equation (26)

$$J = \frac{e}{4\pi\hbar}\sum_{m,j,p,q=1}^{n}\int \mathrm{d}\epsilon\,(f_L(\epsilon) - f_R(\epsilon))[\Gamma^L_{jm}(\epsilon)G^r_{jp}(\epsilon)\Gamma^R_{pq}(\epsilon)G^a_{qm}(\epsilon) + \Gamma^R_{jm}(\epsilon)G^r_{jp}(\epsilon)\Gamma^L_{pq}(\epsilon)G^a_{qm}(\epsilon)]. \tag{42}$$

If the coupling coefficients between the leads and the two quantum dots at the end of the chain are equal, we have $\Gamma^L_{jm} = \Gamma^R_{jm}$ and the current then becomes

$$J = \frac{e}{h}\int \mathrm{d}\epsilon\,(f_L(\epsilon) - f_R(\epsilon))\mathbb{I}(\epsilon) \tag{43}$$

where $\mathbb{I}(\epsilon) = \sum^n_{m,j,p,q=1}\Gamma^L_{jm}(\epsilon)G^r_{jp}(\epsilon)\Gamma^R_{pq}(\epsilon)G^a_{qm}(\epsilon)$. Equation (43) coincides with the Laudauer formula in the non-interacting case [12]. If there are two quantum dots instead of

the N dots, equation (43) coincides with [14], except that we do not consider the effect of the electronic spin. Defining the conductance difference $\lambda_n = \frac{\langle I_n \rangle}{\Delta V}$, we have in the linear response

$$\lambda_n = -\frac{e^2}{h} \int_{-\frac{\mu}{KT}}^{\infty} \mathrm{d}x \, \mathbb{I}'(x) \frac{\mathrm{d}f\,(x)}{\mathrm{d}x}$$

with $x = (\epsilon - \mu)/KT$ and $\mathbb{I}'(x) = \mathbb{I}(\epsilon)$. It is seen that this expression of the conductance has the same form as that derived from the Landauer theory, in which $\mathbb{I}(\epsilon)$ corresponds to the transmission coefficient. It should be noted that the Laudauer theory is suitable for non-interaction between the particles, so the transmission coefficient in the Laudauer formula is determined only by the boundary condition between the reservoirs and the transport wire, but in our case, due to the interaction having been considered, the transmission coefficient depends not only on the boundary of the system but also on the interaction between the particles.

5. Conclusion

We derive the current in the chain of the quantum dots in the static case. Due to the four-fermion interactive terms in the Hubbard model having been neglected, the chain can be decoupled to a series of isolated quasi-dots arranged in a parallel form, which couple with the reservoirs. Because the system is in non-equilibrium, we adopt the non-equilibrium Green function technique to calculate the current. It is seen that the results of the current and the conductance coincide with the Laudauer formula in form, but the transmission coefficient in our case depends on the interaction between the particles. If the spin index is included (where the spin does not take part in the interaction and it only contributes to the number of the transport channel), the current and the conductance are double our above results.

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